IERG 5254: Network Information Theory
Homework 6

(Interference channels)

Due: May 10, 2025

1. Consider an intertference channel with $Y_1 = X_1$. The channel from $(X_1, X_2) \mapsto Y_2$ is arbitrary. Show that the Han-Kobayashi achievable region reduces to the set of rate-pairs satisfying

$$\begin{aligned} R_1 &\leq H(X_1|Q) \\ R_1 + R_2 &\leq I(U_1, X_2; Y_2|Q) + H(X_1|U_1, Q) \\ R_2 &\leq I(X_2; Y_2|U_1, Q) \end{aligned}$$

   for some $p_Q p_{U_1, X_1|Q} p_{X_2|Q}$.

2. Consider an interference channel obtained as follows. Let X_1, X_2 be binary random variables taking values in $\{0, 1\}$. Let $\oplus$ denote the sum (modulo 2). Let $\hat{Y}_1 = X_1 \oplus X_2$ and $\hat{Y}_2 = X_2$. The real receivers Y_1 and Y_2 are obtained by passing $\hat{Y}_1$ and $\hat{Y}_2$ through symmetric BECs, with erasure probabilities ϵ_1 and ϵ_2 respectively. Find the capacity region of this interference channel.

3. Consider an interference channel, with all alphabets binary $\{0, 1\}$, determined by $Y_1 = X_1 \oplus X_2 \oplus Z_1$ and $Y_2 = X_1 \oplus X_2 \oplus Z_2$, where $Z_1 \sim Ber(p_1)$ and $Z_2 \sim Ber(p_2)$ are independent of X_1 and X_2. W.l.o.g. Assume that $0 < p_1 < p_2 < \frac{1}{2}$. Find the capacity region of the interference channel.

4. Consider an interference channel where $Y_1 = y_1(t_2, x_1)$ where $T_2 = f(X_2)$ and further there is a function such that $t_2 = u(y_1, x_1)$. One is also given that $I(X_1; Y_2|X_2) \geq I(X_1; Y_1|T_2)$ for every $p_1(x_1)p_2(x_2)$. Determine the capacity region.