Midterm Examination I

Tuesday, 6 October 2026

Duration: 90 minutes

Name: _____ Student ID: _____

Instructions. Answer all four questions (40 points). Justify every answer; unsupported classifications receive no credit. You may use Fourier-series differentiation and Parseval's identity. State clearly when a derivative is interpreted in the distributional sense.

Question:	System properties	A causal LTI system	Fourier series	Short proof	Total
Points:	12	12	14	2	40
Score:					

Question 1: System properties ..... **[12 points]**

For each system, determine whether it is memoryless, time invariant, linear, causal, BIBO stable, and invertible. Justify *each* classification. Inputs are arbitrary **bounded** real-valued signals on the stated two-sided time axis (not necessarily continuous). Invertibility means injectivity on this input space.

(a) For all integers n, let [6]

$$y[n] = x[n] + 2x[n-1].$$

(b) For all real t, let [6]

$$y(t) = \frac{t}{1+t^2}\, x(t+1).$$

Question 2: A causal LTI system ..... **[12 points]**

A continuous-time LTI system has impulse response

$$h(t) = (\mathrm{e}^{-t} - \mathrm{e}^{-2t})u(t),$$

where $u(t) = 0$ for $t < 0$ and $u(t) = 1$ for $t > 0$. Point values of ordinary functions at isolated discontinuities do not affect convolution.

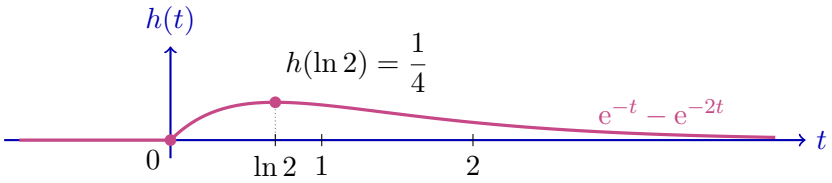


Figure: impulse response $h(t)$.

(a) Compute the output for input $x_1(t) = u(t) - u(t-1)$. Give a piecewise expression for your answer. [4]

(b) Determine the output for $x_2(t) = \delta(t) - \delta(t-2)$. [3]

(c) Interpreting derivatives distributionally, show that [3]

$$h''(t) + 3h'(t) + 2h(t) = \delta(t).$$

Deduce a differential equation relating the output $y = h * x$ to the input x.

(d) Determine whether the system is BIBO stable, and compute $\int_0^\infty h(t)\,\mathrm{d}t$. [2]

Question 3: Fourier series ..... **[14 points]**

Let $x(t) = \sin(\pi t)$ for $0 < t < 1$, extended periodically with period 1. Write its complex Fourier series as $x(t) = \sum_{k\in\mathbb{Z}} c_k \mathrm{e}^{j2\pi kt}$.

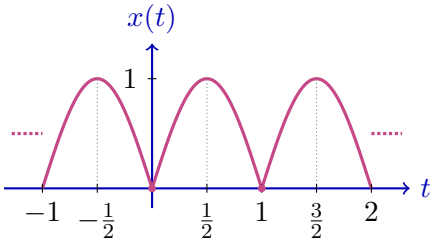


Figure: the period-one extension of $\sin(\pi t)$ *on* $0 < t < 1$.

(a) Find c_k for all integers k by direct integration. [4]

(b) Derive the same formula from the distributional second derivative of the periodic extension. Account for what happens at the integers. [4]

(c) Evaluate the Fourier series at $t = 0$ to find $\sum_{k=1}^{\infty}(4k^2-1)^{-1}$. [3]

(d) Use Parseval to find $\sum_{k=1}^{\infty}(4k^2-1)^{-2}$. [3]

Question 4: Short proof ..... **[2 points]**

Suppose F is an invertible time-invariant system on a shift-invariant signal space, and its inverse is defined on the range of F. Show that the inverse system is time invariant.