MIEG 2051 & ESTR 2360

Signals and Systems

Problem Set 1

Due: Friday, 18 September 2026, 7:00 p.m.

Submission and collaboration.

- Submit a handwritten solution to Gradescope.
- Problems 1–5 will be graded. Problems 6–7 are compulsory for Elite students.
- You may discuss the problems in Piazza after the due date. Discussion of related concepts is allowed in Piazza before the due date, but your submitted work must be your own.
- Unless stated otherwise, include appropriate sketches/plots in your solutions.

The unit-step function is defined by

$$u(t) = \begin{cases} 0, & t < 0, \\ \frac{1}{2}, & t = 0, \\ 1, & t > 0. \end{cases}$$

Problem 1. Find the even and odd decompositions of each signal, and plot its even and odd components.

(a)
$$x(t) = \begin{cases} t, & 0 \le t \le 7, \\ 0, & \text{otherwise.} \end{cases}$$

(b) $x(t) = 4\mathrm{e}^{-2t}u(t)$.

(c) $x[n] = \dfrac{\sin(\pi n)}{\pi n}$, where $x[0] = 1$.

Problem 2. Determine whether each signal is periodic. If it is periodic, determine its fundamental period.

(a)
$$x(t) = \begin{cases} \cos\left(t + \frac{\pi}{4}\right), & t \le 0, \\ \sin\left(t + \frac{\pi}{4}\right), & t \ge 0. \end{cases}$$

(b) $x[n] = \sin\left(\frac{6\pi}{7}n + 1\right)$.

(c) $x[n] = \cos\left(\frac{\pi n}{2}\right) + \cos\left(\frac{\pi n}{4}\right)$.

(d) Consider

$$x_1[n] = \frac{1}{2}\cos\left(\frac{(\pi+2)n}{2}\right), \qquad x_2[n] = \cos\left(\frac{(\pi+1)n}{2}\right)\cos\left(\frac{n}{2}\right),$$

and $x[n] = x_1[n] - x_2[n]$. Determine periodicity for $x_1[n]$, $x_2[n]$, and $x[n]$.

Problem 3. *Frequently used signals.*

(a) **Unit impulse.** For every function $f(t)$ continuous at t_0,

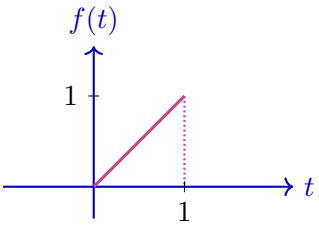
$$\int_{-\infty}^{\infty} f(t)\delta(t-t_0)\,\mathrm{d}t = f(t_0).$$

Compute:

(i) $\displaystyle\int_{-\infty}^{\infty} \mathrm{e}^{-at}\delta(t-5)\,\mathrm{d}t.$

(ii) $\displaystyle\int_{-\infty}^{\infty} \cos(2\pi f_0 t)\delta\left(t+\frac{1}{f_0}\right)\,\mathrm{d}t.$

(b) **Unit step.** Consider the signal shown below.



(i) Use $u(t)$ and shifted versions of $u(t)$ to write a mathematical expression for $f(t)$. (Hint: use integration.)

(ii) Compute and plot $\dfrac{\mathrm{d}f(t)}{\mathrm{d}t}$. Recall that, in this course, differentiation of a discontinuous function produces impulses; for example, $\dfrac{\mathrm{d}}{\mathrm{d}t}u(t) = \delta(t)$.

(c) **Discrete-time step and impulse.** Let $x[n] = a^n u[n]$.

(i) Find and plot $g[n] = x[n] - ax[n-1]$.

(ii) Use $g[n]$ to express $x[n]$.

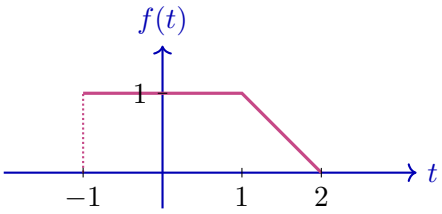
Problem 4. *Properties of systems.* For each system, determine whether it is linear, causal, time-invariant, memoryless, invertible, and stable. In each case, justify your answer from the definitions.

(a) $y(t) = \mathrm{e}^{x(t)}$.

(b) $y[n] = x[n]x[n-1]$.

(c) $y[n] = x[2n]$.

Problem 5. Given $f(t)$ shown below, plot $g(t) = 2f(5-2t)$ by using the indicated intermediate steps.



Plot the intermediate signals $f_1(t)$, $f_2(t)$, $f_3(t)$, and finally $g(t)$:

(a) $f_1(t) = f(t+5)$.

(b) $f_2(t) = f_1(-t) = f(-t+5)$.

(c) $f_3(t) = f_2(2t) = f(-2t+5)$.

(d) $g(t) = 2f_3(t) = 2f(5-2t)$.

Question to think about (do not include this in your submitted solution): Can the operations be performed in other orders? How must the shift be adjusted when it is moved past reflection or time scaling?

Problem 6. Elite problem. Suppose a continuous signal $x(t)$ has periods $T_1 = 1$ and $T_2 = \sqrt{2}$. Show that $x(t)$ must be constant.

Problem 7. Elite problem. Write the signal $y(t)$ as a linear combination of the signal $x(t)$ and its time shifts, where $x(t)$ and $y(t)$ are shown below.

