MIEG 2051 & ESTR 2360

Signals and Systems

Problem Set 2

Due: Friday, 25 September 2026, 7:00 p.m.

Submission and collaboration.

- Submit a handwritten solution to Gradescope.
- Problems 1–5 will be graded. Problem 6 is compulsory for Elite students.
- You may discuss the problems in Piazza after the due date. Discussion of related concepts is allowed in Piazza before the due date, but your submitted work must be your own.
- Unless stated otherwise, include appropriate sketches/plots in your solutions.

Throughout this problem set, the unit-step function is

$$u(t) = \begin{cases} 0, & t < 0, \\ \frac{1}{2}, & t = 0, \\ 1, & t > 0. \end{cases}$$

Problem 1. *Properties of systems.* For each statement, state whether it is **true** or **false**. Briefly justify your answer.

(a) Every memoryless system is causal.

(b) Every causal system is memoryless.

(c) A system is invertible if and only if distinct inputs always produce distinct outputs.

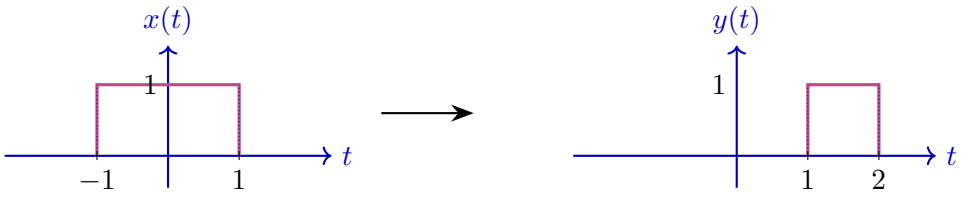
(d) A system is time invariant if a time shift in the input produces no change in the output.

(e) A linear system can produce a nonzero output when its input is identically zero.

Problem 2. *Understanding LTI-system behaviour.* An LTI system maps the input $x(t)$ to the output $y(t)$ shown below. Thus

$$x(t) = \operatorname{rect}\left(\frac{t}{2}\right), \qquad y(t) = \operatorname{rect}\left(\frac{t - \frac{3}{2}}{1}\right).$$

Values at isolated endpoints are irrelevant for this question.



Compute the output of the same system for each of the following inputs.

(a) $x_1(t) = x(t+3) + 2x(t+2)$.

(b) $x_2(t) = u(t)$. Hint: express $u(t)$ as a sum of appropriately shifted copies of $x(t)$.

(c) $x_3(t) = \delta(t)$. Hint: differentiate the result from part (b).

Problem 3. *An exercise in convolution.* Define

$$\operatorname{rect}(t) = \begin{cases} 1, & |t| < \frac{1}{2}, \\ \frac{1}{2}, & |t| = \frac{1}{2}, \\ 0, & |t| > \frac{1}{2}, \end{cases} \qquad \operatorname{tri}(t) = \begin{cases} 1 - |t|, & |t| < 1, \\ 0, & |t| \geq 1. \end{cases}$$

Show that, for $a > 0$,

$$\frac{1}{a}\operatorname{rect}\left(\frac{t}{a}\right) * \frac{1}{a}\operatorname{rect}\left(\frac{t}{a}\right) = \frac{1}{a}\operatorname{tri}\left(\frac{t}{a}\right).$$

You may use either a direct integral calculation or a geometric overlap argument.

Problem 4. Let

$$x(t) = \sum_{\ell=2}^{\infty} \delta(t-\ell), \qquad h(t) = \begin{cases} 1, & |t| \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Find and sketch the output $y(t) = x(t) * h(t)$.

Problem 5. *Area and centre-of-gravity theorems.* Let $y(t) = x(t) * h(t)$, and assume that all integrals below exist absolutely.

(a) Show that

$$\int_{-\infty}^{\infty} y(t)\,\mathrm{d}t = \left(\int_{-\infty}^{\infty} x(t)\,\mathrm{d}t\right)\left(\int_{-\infty}^{\infty} h(t)\,\mathrm{d}t\right).$$

(b) Assuming that the three integrals in the denominators are nonzero, show that

$$\frac{\int_{-\infty}^{\infty} ty(t)\,\mathrm{d}t}{\int_{-\infty}^{\infty} y(t)\,\mathrm{d}t} = \frac{\int_{-\infty}^{\infty} tx(t)\,\mathrm{d}t}{\int_{-\infty}^{\infty} x(t)\,\mathrm{d}t} + \frac{\int_{-\infty}^{\infty} th(t)\,\mathrm{d}t}{\int_{-\infty}^{\infty} h(t)\,\mathrm{d}t}.$$

Problem 6. *Linear systems — compulsory for Elite students.* A linear system, which is *not* time invariant, has output $h_{t_0}(t)$ when its input is $\delta(t - t_0)$, where

$$h_{t_0}(t) = \begin{cases} 1, & |t| < \dfrac{|t_0|}{2}, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Write an expression for the output $y(t)$ for a general input $x(t)$.

(b) Compute and sketch the output when

$$x(t) = \begin{cases} 1, & |t| < 2, \\ 0, & \text{otherwise.} \end{cases}$$

(c) For the input in part (b), find the output when the input is replaced by $x'(t)$.

(d) Show that, for a general input $x(t)$ satisfying $x(t) \to 0$ as $|t| \to \infty$, the output corresponding to $x'(t)$ is

$$x(-2|t|) - x(2|t|).$$