MIEG 2051 & ESTR 2360

Signals and Systems

Problem Set 3

Due: Friday, 2 October 2026, 7:00 p.m.

Submission and collaboration.

- Submit a handwritten solution to Gradescope.
- Problems 1–3 will be graded. Problem 4 is compulsory for Elite students.
- You may discuss the problems in Piazza after the due date. Discussion of related concepts is allowed in Piazza before the due date, but your submitted work must be your own.
- Please justify your answers. Include a sketch when requested.

For a signal with stated period T, use the convention

$$x(t) = \sum_{k\in\mathbb{Z}} c_k \mathrm{e}^{j2\pi kt/T}, \qquad c_k = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\mathrm{e}^{-j2\pi kt/T}\,\mathrm{d}t.$$

The specified values at isolated endpoints do not affect Fourier coefficients. In Problem 1, extend each defining formula periodically using the indicated interval.

Problem 1. *Related Fourier series.* Calculate the complex Fourier coefficients in each part. Use relationships among the signals where convenient; state the period you use for each coefficient sequence.

(a) $x^{(a)}(t) = 0$ for $-1 < t < 0$ and $x^{(a)}(t) = t$ for $0 < t < 1$, extended with period 2. Call its coefficients a_k.

(b) $x^{(b)}(t) = t$ for $-1 < t < 1$, extended with period 2. Call its coefficients b_k.

(c) $x^{(c)}(t) = t + 1$ for $-1 < t < 1$, extended with period 2.

(d) $x^{(d)}(t) = t$ for $0 < t < 2$, extended with period 2.

(e) $x^{(e)}(t) = 2t$ for $0 < t < 1$, extended with period 1.

(f) $x^{(f)}(t) = |t|$ for $-1 < t < 1$, extended with period 2.

(g) Let $x^{(g)}(t) = x^{(a)}(2t)$. Find its coefficients first when represented with period 1 and then when represented with period 2. Explain why the two sequences have different indices but describe the same signal.

Problem 2. *Periodic convolution and a sketch.* Let x_1 and x_2 be period-1 signals with Fourier expansions

$$x_1(t) = \sum_{k\in\mathbb{Z}} a_k \mathrm{e}^{j2\pi kt}, \qquad x_2(t) = \sum_{k\in\mathbb{Z}} b_k \mathrm{e}^{j2\pi kt}.$$

Assume x_1 is real valued, $a_1 = a_{-1} > 0$, and

$$b_k = \begin{cases} 1/a_k, & |k| = 1, \\ 0, & \text{otherwise.} \end{cases}$$

Define $y(t) = \int_{-1/2}^{1/2} x_1(\tau) x_2(t-\tau)\,\mathrm{d}\tau$. Compute $y(t)$ and sketch one period, marking its maxima, minima, and zeros. *Hint:* What happens to Fourier coefficients under periodic convolution?

Problem 3. *High-frequency Fourier coefficients.* Represent each signal using period $T = 1$:

$$x(t) = \sum_{k\in\mathbb{Z}} c_k \mathrm{e}^{j2\pi kt}.$$

Compute all its nonzero coefficients when

(a) $x(t) = \cos(50\pi t)$;

(b) $x(t) = \cos(50\pi t - \pi/4)$.

For part (b), check that your answer satisfies the conjugate-symmetry property of a real-valued signal.

Problem 4. Elite problem: Cauchy–Schwarz and the triangle inequality. Let V be a complex inner-product space with inner product linear in the *first* argument. Recall $\langle x, y\rangle = \overline{\langle y, x\rangle}$ and $\langle x, x\rangle > 0$ for $x \neq 0$. Define $\|x\| = \sqrt{\langle x, x\rangle}$.

(a) Prove Cauchy–Schwarz: $|\langle x, y\rangle| \leq \|x\|\|y\|$. If $y \neq 0$, consider $\langle x - \alpha y, x - \alpha y\rangle$ with $\alpha = \langle x, y\rangle / \langle y, y\rangle$. Deal separately with $y = 0$.

(b) Prove the triangle inequality: $\|x + y\| \leq \|x\| + \|y\|$.