

MIEG 2051 & ESTR 2360 Fourier Analysis with Engineering Applications

Problem Set 1

Due: September 12, 2025 (11:30 P.M.)

1. Homework should be submitted via Gradescope.
2. Only problems 1,2,3,4,5 will be graded. Problem 6 and 7 are compulsory for Elite Students
3. You may discuss the problems in Piazza after the due date.
4. Discussion of related concepts is allowed in Piazza.
5. The homework must be hand-written.

The unit step function $u(t)$ is defined according to

$$u(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{2} & t = 0 \\ 1 & t > 0 \end{cases}.$$

1. Find the even and odd decomposition for the following signals and plot the even and odd signals

(a) $x(t) = \begin{cases} t, & 0 \leq t \leq 7 \\ 0, & \text{otherwise} \end{cases}$

(b) $x(t) = 4e^{-2t}u(t)$. Here $u(t)$ is the unit-step function defined above.

(c) $x[n] = \frac{1}{\pi n} \sin(\pi n)$. Take $x[0] = 1$.

2. Are the following signals periodic? If yes, determine the fundamental period.

(a) $x(t) = \begin{cases} \cos(t + \frac{\pi}{4}), & t \leq 0 \\ \sin(t + \frac{\pi}{4}), & t \geq 0 \end{cases}$

(b) $x[n] = \sin(\frac{6\pi}{7}n + 1)$

(c) $x[n] = \cos \frac{\pi n}{2} + \cos \frac{\pi n}{4}$

(d) (i) $x_1[n] = \frac{1}{2} \cos \frac{(\pi+2)n}{2}$

- (ii) $x_2[n] = \cos \frac{(\pi+1)n}{2} \cos \frac{n}{2}$
 (iii) $x[n] = x_1[n] - x_2[n] = \frac{1}{2} \cos \frac{(\pi+2)n}{2} - \cos \frac{(\pi+1)n}{2} \cos \frac{n}{2}$

3. Frequently used signals

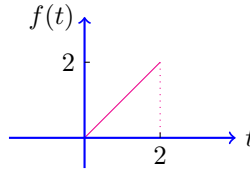
- (a) Unit impulse function $\delta(t)$.

Defintion: Any function $f(t)$ that is continuous at t_0 obeys

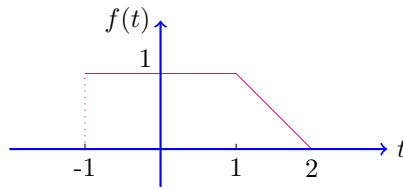
$$\int_{-\infty}^{+\infty} f(t)\delta(t-t_0)dt = f(t_0).$$

Compute the following integrals

- (i) $\int_{-\infty}^{+\infty} e^{-at}\delta(t-5)dt$
 (ii) $\int_{-\infty}^{+\infty} \cos(2\pi f_0 t)\delta(t + \frac{1}{f_0})dt$
 (b) Unit step function $u(t)$. Consider $f(t)$ in the following figure



- (i) Use $u(t)$ (and shifted versions) to write a mathematical expression (hint: using integration) for $f(t)$
 (ii) Compute and plot the derivative $\frac{df(t)}{dt}$ (Remark: In this class, differentiation of functions that are not continuous produces δ -functions. For instance $\frac{d}{dt}u(t) = \delta(t)$.)
 (c) Discrete cases: $u[n]$ and $\delta[n]$. Consider the signal $x[n] = a^n u[n]$
 (i) Find and plot signal $g[n] = x[n] - ax[n-1]$
 (ii) Use signal $g[n]$ to express $x[n]$
4. *Properties of systems:* Determine if each of the system is: linear, causal, time-invariant, memoryless, invertible, or stable.
- (a) $y(t) = e^{x(t)}$
 (b) $y[n] = x[n]x[n-1]$
 (c) $y[n] = x[2n]$
5. Given $f(t)$ shown below, plot $g(t) = 2f(5-2t)$ by using the following intermediate steps: Plot each of the intermediate figures $f_1(t)$, $f_2(t)$, $f_3(t)$ and finally $g(t)$.
- (a) Perform a shift to obtain $f_1(t) = f(t+5)$.
 (b) Perform a reflection to obtain $f_2(t) = f_1(-t) = f(-t+5)$.
 (c) Perform a scaling of time to obtain $f_3(t) = f_2(2t) = f_1(-2t) = f(-2t+5)$.



- (d) Finally perform the amplification to obtain $g(t) = 2f_3(t) = 2f_2(2t) = 2f_1(-2t) = 2f(5 - 2t)$.

Questions to think about [no need to include in your solutions]: Can I also do this in a different orders? If so, what other orders. Can one say something more about the various possibilities; e.g. the location of the shifting operation w.r.t to reflection and scaling?

6. Extra problem (Only **required** for Elite students).

Suppose a continuous signal $x(t)$ has periods $T = 1$ and $T = \sqrt{2}$. Then argue that $x(t)$ must be a constant.

7. Extra problem (Only **required** for Elite students).

Write the signal $y(t)$ as a linear combination of the signal $x(t)$ and its time shifts.

