

MIEG 2051 & ESTR 2360
Signals and Systems
Problem Set 2

Due: Sep 19, 2025 (11:30 P.M.)

1. Please submit your homework on gradescope
2. Only problems 1,2,3,4,5 will be graded. Problem 6 is compulsory for Elite Students
3. You can discuss Problems 1-5 within your group members.
4. You may discuss the problems in Piazza after the due date.
5. Discussion of related concepts is allowed in Piazza.
6. The homework must be hand-written.

1. *Properties of systems:* For each statement, state if they are **true** or **false**:
 - (a) A memoryless system is definitely a causal system.
 - (b) A causal system is definitely a memoryless system.
 - (c) A system is invertible if distinct inputs lead to distinct outputs.
 - (d) A system is time invariant if a time shift in the input signal produces no change in the output signal.
 - (e) Given a zero input to a linear system, it is possible to have nonzero output.
2. *Understanding LTI system behavior:* Consider an LTI system that maps an input $x(t)$ to the output $y(t)$ as shown in the Figure 1. (If one is worried of values of $x(t)$ at $t = -1, 1$, take them to be $\frac{1}{2}$. In principle, the exact value at these points do not matter.)

Compute the output of the same LTI system for the following three inputs:

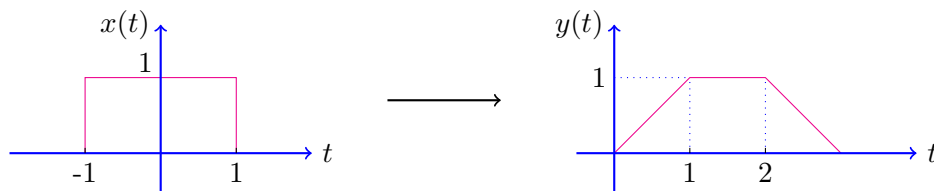


Figure 1: Input and output of an LTI system

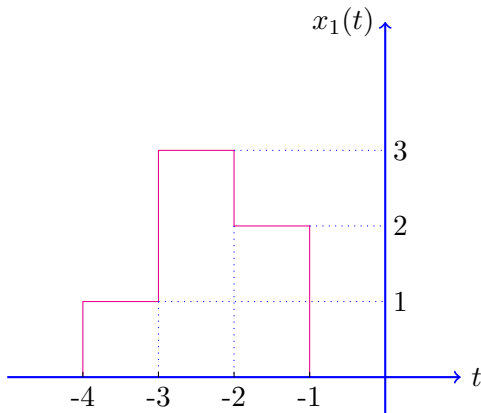


Figure 2: Input $x_1(t)$

- (a) Input $x_1(t)$ shown in Figure 2. (Hint: try to express $x_1(t)$ as a linear combination of $x(t+3)$ and $x(t+2)$.)
- (b) Input $x_2(t) = u(t)$, where $u(t)$ is the unit step function defined in the notes. (Hint: Consider $\sum_{m=0}^{\infty} x(t-2m-1)$)
- (c) Input $x_3(t) = \delta(t)$, where $\delta(t)$ is the delta function. (Hint: use the above part and the differentiation property of convolution.)

3. *An exercise in convolution:* Let

$$\text{Rect}(t) = \begin{cases} 1 & |t| < \frac{1}{2} \\ \frac{1}{2} & |t| = \frac{1}{2} \\ 0 & |t| > \frac{1}{2} \end{cases} \quad \text{and} \quad \text{Tri}(t) = \begin{cases} 1 - |t| & |t| < 1 \\ 0 & |t| \geq 1 \end{cases}.$$

Show that: $\frac{1}{a}\text{Rect}(\frac{t}{a}) * \frac{1}{a}\text{Rect}(\frac{t}{a}) = \frac{1}{a}\text{Tri}(\frac{t}{a})$. (Assume $a > 0$)

(Remark: Integrations in the convolution can be done either algebraically or geometrically using pictures.)

4. Let $x(t) = \sum_{l=2}^{\infty} \delta(t-l)$ be the input, and $h(t) = \begin{cases} 1 & |t| \leq 1 \\ 0 & \text{otherwise} \end{cases}$ be the impulse response.

Question: What's the output $y(t) = x(t) * h(t)$?

5. *Area and center of gravity theorems*: Let $y(t) = x(t) * h(t)$. Show that

(a)

$$\int_{-\infty}^{\infty} y(t) dt = \left(\int_{-\infty}^{\infty} x(t) dt \right) \left(\int_{-\infty}^{\infty} h(t) dt \right)$$

(b)

$$\frac{\int_{-\infty}^{\infty} ty(t) dt}{\int_{-\infty}^{\infty} y(t) dt} = \left(\frac{\int_{-\infty}^{\infty} tx(t) dt}{\int_{-\infty}^{\infty} x(t) dt} \right) + \left(\frac{\int_{-\infty}^{\infty} th(t) dt}{\int_{-\infty}^{\infty} h(t) dt} \right).$$

(For the second part, assume that $\int_{-\infty}^{\infty} y(t) dt, \int_{-\infty}^{\infty} x(t) dt, \int_{-\infty}^{\infty} h(t) dt$ are non-zero.)

6. *Linear Systems (Midterm 2012) - Compulsory for Elite students*

Consider a linear system (this system is *not time-invariant*) which has the following output $h_{t_0}(t)$ (shown in Figure 3) when the input is $\delta(t - t_0)$.

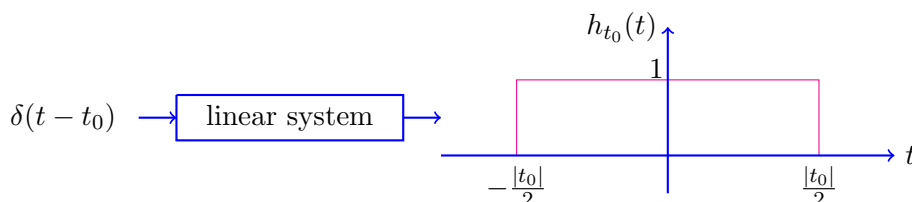


Figure 3: The system behavior for shifted δ functions

- Write an expression for the output $y(t)$ of the system in terms of the input $x(t)$ to the system. (Here $x(t)$ is a generic input signal). (Hint: Express $x(t)$ as a linear combination of the shifted impulses $\delta(t - t_0)$.)
- Compute and plot the output when the input $x(t)$ is as shown below in Figure 4.
- What is the output of the system when the input $x_1(t) = \frac{dx(t)}{dt}$, where $x(t)$ is the signal shown in Figure 4?
- Show that the output of $\frac{dx(t)}{dt}$, for any generic $x(t)$, is given by $x(-2|t|) - x(2|t|)$. (Assume that $x(t) \rightarrow 0$ as $|t| \rightarrow \infty$.)

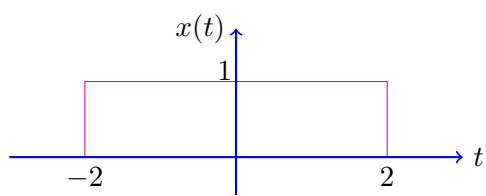


Figure 4: Input $x(t)$