

IERG 2051 B & ESTR 2360
Signals and Systems
Problem Set 3

Due: Sep 26, 2025 (11:30 P.M.)

1. Please submit your homework in Gradescope
2. Only problems 1, 2, and 3 will be graded. Problem 4 is compulsory for Elite Students.
3. You may discuss the problems in Piazza after the due date.
4. Discussion of related concepts is allowed in Piazza.
5. The homework must be hand-written.
6. Please justify your answers.

1. Calculate the Fourier series with the given period interval (Hint: Use properties of Fourier-Series to easily deduce the answers from earlier parts.)

(a) $x^a(t) = \begin{cases} 0, & t \in [-1, 0] \\ t, & t \in [0, 1] \end{cases}.$

(b) $x^b(t) = t, [-1, 1]$

(c) $x^c(t) = t + 1, [-1, 1]$

(d) $x^d(t) = t, [0, 2]$

(e) $x^e(t) = 2t, [0, 1]$

(f) $x^f(t) = |t|, [-1, 1]$

(g) $x^g(t) = x^a(2t)$

- Do this part assuming period remains same, i.e. $T = 2$
- Do this part with the new period, i.e. $T = 1$.

2. Let $x_1(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt}$ and let $x_2(t) = \sum_{k=-\infty}^{\infty} b_k e^{j2\pi kt}$. Here assume that $a_k > 0 \forall k$ and that $b_k = \frac{k}{a_k}$, when $|k| = 1$ and $b_k = 0$ otherwise. Further, define $y(t) = \int_{-1/2}^{1/2} x_1(\tau)x_2(t - \tau)d\tau$.

Compute and plot $y(t)$. [Hint: Use orthogonality.]

3. Assume $T_0 = 1$, i.e. express $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt}$. Compute a_k when

(a) $x(t) = \cos(50\pi t)$

(b) $x(t) = \cos(50\pi t - \pi/4)$

4. **Cauchy-Schwartz inequality and Triangle inequality:** Consider a vector space V over complex numbers (Vector space is a collection of objects closed under addition and scalar multiplication). An *inner product* is a function defined $\langle \cdot, \cdot \rangle : V \times V \mapsto \mathbb{C}$ that satisfies the following properties:

- $\langle ax_1 + bx_2, y \rangle = a\langle x_1, y \rangle + b\langle x_2, y \rangle$. The operation is linear in the first variable.
- $\langle x, y \rangle = \langle y, x \rangle^*$ where a^* denotes the complex conjugation.
- $\langle x, x \rangle \geq 0$ with equality *if and only if* $x = 0$. This property is called positive definiteness.
- **Prove** the following inequality for any inner product (also called Cauchy-Schwartz inequality).

$$|\langle x, y \rangle| \leq \langle x, x \rangle^{\frac{1}{2}} \langle y, y \rangle^{\frac{1}{2}}.$$

(Hint: consider $\langle x - ay, x - ay \rangle$ where $a = \frac{\langle x, y \rangle}{\langle y, y \rangle}$.)

In an inner product space, the length (or norm) of a vector is usually computed as

$$\|x\| = \langle x, x \rangle^{\frac{1}{2}}.$$

- Show that the lengths of vectors satisfy *the triangle inequality*, i.e.

$$\|x + y\| \leq \|x\| + \|y\|.$$