

MIEG 2051 & ESTR 2360
Signals and Systems
Problem Set 4

Due: October 13, 2025

1. *Eigenfunctions of LTI systems:*

- (a) Given an LTI system with impulse response $h(t) = \delta(t)$, what are the eigenfunction(s) and the associated eigenvalue(s) of this LTI system?
- (b) Given an LTI system with impulse response $h(t)$, show that both $\cos(\omega t)$ and $\sin(\omega t)$ are eigenfunction(s) of this LTI system when $h(t)$ is an even function.
- (c) Given an LTI system with impulse response $h(t)$, where

$$\hat{H}(f) = \begin{cases} \frac{1}{2}(1 - e^{-j2\pi f/f_c}) & -f_c \leq f \leq f_c \\ 0 & \text{otherwise} \end{cases}$$

Find the output $y(t)$ of the LTI system when the input $x(t) = \cos(\pi f_c t) + \sin(4\pi f_c t)$.

2. *Basic FT Calculation:*

- (a) Find the Fourier transform of the following signals: (Hint: sometimes it may be easier to use the inverse transform to show the relationship.)

- (i) $x_1(t) = e^{-a|t|}$, where the constant $a > 0$.

- (ii) $x_2(t) = e^{-|t|} \sin(t)$.

- (iii) $x_3(t) = \frac{\sin t}{t}$.

Also calculate $\int_{-\infty}^{+\infty} \frac{\sin t}{t} dt$. (Hint: express $\int_{-\infty}^{+\infty} \frac{\sin t}{t} dt$ in terms of F.T. of $x_3(t)$.)

- (b) Let $\hat{X}(f)$ be the Fourier transform of $x(t)$ which is given by

$$x(t) = \begin{cases} 0 & t < -1/2 \\ \frac{1}{2} - |t| & -1/2 \leq t \leq 1/2 \\ 0 & t > 1/2 \end{cases}$$

- (i) Find $\hat{X}(0)$.
 - (ii) Find $\int_{-\infty}^{+\infty} \hat{X}(f) df$.
 - (iii) Find $\int_{-\infty}^{+\infty} |\hat{X}(f)|^2 df$.
 - (iv) Sketch the inverse Fourier transform of the real part of $\hat{X}(f)$.
- (Remark: you can do all these parts by using the properties of F.T.)

3. *FT Properties:*

Given $x(t)$ and its Fourier transform $\hat{X}(f)$:

- (a) show that $x(t)$ is a real signal if and only if $\hat{X}^*(f) = \hat{X}(-f)$.
- (b) find the Fourier transform of $x(6 - 2t)$.
- (c) If $y(t) = \int_{-\infty}^t x(2(\tau - 1)) d\tau$, find the Fourier transform of $y(t)$.
- (d) Let, $x(t) = e^{-t}u(t)$ and $h(t) = e^t u(-t)$. Calculate $y(t) = x(t) * h(t)$. (Hint: Use convolution property of FT.)

4. *FT of Periodic Functions:*

Find the Fourier transform of the periodic function $y(t) = \sum_{n=-\infty}^{\infty} x(t - nT)$ where $x(t) = \delta(t) - \delta(t - T/2)$.

5. *FT using tables:*

Find the Fourier transform of

$$x(t) = \begin{cases} t & t \in [0, 1] \\ 1 & t \in [1, 2] \\ 3 - t & t \in [2, 3] \\ 0 & \text{otherwise} \end{cases}.$$

(Hint: Use tables cleverly. There are at least 2 different ways of doing this question without performing a single integral but rather using properties and transforms of known signals; and a third direct way of computing the integral. It would be useful to try to do it in all ways you can imagine to get additional practice.)

6. *Poisson summation formula - Compulsory for Elite students*

Let $g(t)$ be a real-valued function and $\hat{G}(f)$ be its Fourier Transform. The show that

$$\sum_{k \in \mathbb{Z}} g(k) = \sum_{k \in \mathbb{Z}} \hat{G}(k)$$

(Hint: Consider $x_s(t) = x(t) (\sum_{n \in \mathbb{Z}} \delta(t - n))$ and its Fourier Transform.)

7. *Contour integration - Compulsory for Elite students*

Use contour integration to compute the Fourier Transform of

$$x(t) = \frac{\sin(\pi t)}{\pi t}.$$