

MIEG 2051 & ESTR 2360
Signals and Systems
Problem Set 6

Due: Nov 14 2025

1. Find the Laplace Transform of the following signals and specify its ROC.

(a) $x(t) = e^{-2t}u(t) + e^{-3t}u(t)$.

(b) $x(t) = te^{-2|t|}$.

(c) $x(t) = \delta(t) + u(t)$.

2. Find the inverse Laplace Transform.

(a) $\hat{X}(s) = \frac{1}{s^2+9}$; $ROC : \mathbf{Re}(s) > 0$

(b) $\hat{X}(s) = \frac{s+1}{s^2+5s+6}$; $ROC : -3 < \mathbf{Re}(s) < -2$

(c) $\hat{X}(s) = \frac{1}{(s+1)(s+2)}$; for all the three possible choices of ROC.

3. Determine whether the following statements are true or false. If true, show a proof; if false, produce a counterexample.

(a) The Laplace transform of $t^2u(t)$ does not converge in any part of the s -plane.

(b) The Laplace transform of $e^{t^2}u(t)$ does not converge in any part of the s -plane.

For the following two parts assume that $h(t)$ corresponds to a stable and causal system with a rational $\hat{H}(s)$.

(a) The impulse response $h_1(t) = \frac{dh(t)}{dt}$ corresponds to a stable and causal system.

(b) The impulse response $h_2(t) = \int_{-\infty}^t h(\tau)d\tau$ corresponds to a stable and causal system.

4. Consider an LTI system characterized by

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 3x(t).$$

- (a) Find the system impulse response $\hat{H}(s)$.
 - (b) Determine $y(t)$ when $x(t) = \delta(t) - 2e^{-3t}u(t)$.
5. A real signal $x(t)$ with a rational Laplace transform denoted by $\hat{X}(s)$ satisfies the following conditions:
- (a) $\hat{X}(s)$ has only two poles.
 - (b) $\hat{X}(s)$ has no zero in the finite s -plane.
 - (c) One pole of $\hat{X}(s)$ is located at $s = -1 + j$.
 - (d) $e^{2t}x(t)$ is not absolutely integrable.
 - (e) $\hat{X}(0) = 8$.

Find $\hat{X}(s)$ and its ROC.

6. **ELITE.** Let $h(t)$ denote the impulse response of a causal and stable system, and further assume that $\lim_{t \rightarrow \infty} h(t)$ exists. Then show that

$$\lim_{\sigma \downarrow 0} \sigma \hat{H}(\sigma) = \lim_{t \rightarrow \infty} h(t).$$